November 1994

A Nonperturbative, Schwinger-Dyson-Equation Analysis of Quark Masses and Mixings in a Model with QCD and Higgs Interactions

Lesley L. Smith[a], Pankaj Jain[b], Douglas W. McKay[a,c]

[a] Department of Physics and Astronomy, University of Kansas, Lawrence, KS 66045

[b] Department of Physics and Astronomy, University of Oklahoma, Norman, OK, 73019

[c] Kansas Institute for Theoretical and Computational Science, University of Kansas, Lawrence, KS 66045

Abstract

The Schwinger-Dyson equation for the quark self-energy is solved in the quenched ladder approximation for several cases of one- and two-quark-generations. The exchanges of standard model gluons and Higgs bosons are taken into account. It is found that Higgs boson exchange dominates the quark self-energy in the ultraviolet region for sufficiently large input quark masses (>75 GeV), causing the running quark propagator mass to increase with energy-scale. The running of the quark mixing angles is also considered. No running of the quark mixing angles is found for input quark masses up to and including 500 GeV.

I. Introduction

Unlike the renormalization group equation, the Schwinger-Dyson equation (SDE) analysis presented here enables us in principle to calculate the running quark mass functions for very light quarks at low energies where quantum chromodynamics (QCD) is in the non-perturbative region. This is of interest because experimentally accessible quantities are presently found at low energy-scales. In the presence of heavy quarks, such as bottom and top, it allows the calculation of running mass matrix at momentum scale of the order of less than the quark masses, at which scale the RG equation results may not be reliable. Moreover, this nonperturbative SDE approach is also interesting in that it can describe the running mass function of an ultra-heavy quark, with a large Yukawa coupling that cannot be treated reliably by perturbation theory. This would be relevant for a fourth generation quark, for example.

The calculation of the variation of mixing angles with momentum scale is of great experimental interest since they are usually measurable only in a very limited kinematic region. To illustrate this point, consider the top-strange CKM mixing parameter, V_{ts}. Direct top-antitop quark bound state production at the next linear collider would occur at approximately $> 300\,\mathrm{GeV}^2$, and V_{ts} would be evaluated there. Presently, some estimations of V_{ts} are being done indirectly using penguin diagrams in the calculation of $b \to s$ transitions. In this method a virtual top quark mixes with a virtual strange quark at a few GeV^2 [1]. Clearly if $V_{ts}(300\ \mathrm{GeV}^2) \neq V_{ts}(\simeq\ 3\ \mathrm{GeV}^2)$, complications would arise. This possibility should be investigated both by RG techniques and by the SDE. In particular SDE studies can reveal the existence of any nonperturbative effects which might be absent in RG studies.

There have been several studies of the quark propagator Schwinger-Dyson Equation. These have fallen into the areas of chiral-symmetry breaking [2, 3, 4, 5, 6], confinement [7], and related phenomenology including pseudoscalar decay constants and form factors [8, 9, 10, 11]. The complexity of the SDEs makes it neccesary to use various approximations or assumptions. The numerous QCD quark SDE studies make assumptions about the quark-gluon vertex and they model the effective gluon propagator. Many of these studies utilize the Landau gauge, ladder approximation and the angle approximation to express the integral

SDE in terms of a differential equation [12]. Recent reviews of SDE literature which include discussion of and references for the above points include [12, 13, 14].

Though the propagator is a gauge dependent object, and this is true in the approximations that we use, our results do agree with the gauge-invariant one-loop renormalization group analysis results in the ultraviolet region. We are careful not to neglect derivatives of the strong coupling, $\alpha_s(q^2)$, to be consistent with the one-loop RGE result [15].

Roberts and McKellar [16] have studied the validity of the angle approximation used in the context of the model of Atkinson and Johnson [5]. They use the Landau gauge and the running strong coupling $\alpha_s(q^2) = d\pi/ln(x_0 + q^2/\Lambda_{QCD}^2)$, and re-express the integral equation as a differential equation. Roberts and McKellar find that the angle approximation is qualitatively reasonable, introducing errors of order 12%. For other models, however, the angle approximation is not so reliable, particularly when the strong coupling becomes singular at the origin. To minimize problems with the angle approximation, we utilize some aspects of the model of Atkinson and Johnson [5].

Rather than investigate chiral symmetry breaking, this study assumes that chiral symmetry is broken and solves the quark propagator SDE to find running quark mass functions and running quark mixing angles. In addition to the usual gluon contributions, we include Higgs boson exchange effects in the quark self-energy. The contributions to the quark self-energy due to Higgs bosons are being studied here for the first time in the context of the quark propagator SDE. We also study for the first time in a Higgs-boson-plus-gluon model the multigenerational cases of quarks which enable us to analyze quark mixing angles.

II. Model

The general form of the quark propagator SDE that we use is

$$S^{-1}(q) = S_0^{-1}(q) - \int \frac{d^4k}{(2\pi)^4} g_s\gamma_\mu S(k) g_s \Lambda_\nu G^{\mu\nu} - \int \frac{d^4k}{(2\pi)^4} g_H S(k) g_Y \Lambda_H P_H, \quad (1)$$

where $g_s\gamma_\mu$ and $g_s\Lambda_\nu$ are quark-gluon vertices, $S(k)$ is the quark propagator, $G^{\mu\nu}$ is the gluon propagator, $g_Y\Lambda_H$ is the quark-Higgs boson vertex factor, and P_H is the Higgs boson propagator, to be discussed below.

In this study we will simplify the quark propagator by using the ladder approximation namely we will set all the vertices to be the bare vertices. We will, however, use the RG improved expressions for both the QCD and Yukawa couplings. In the Landau gauge $A(q) =$ 1 in the approximate form used here of the Schwinger-Dyson-Equation (SDE) in QCD [17] Thus the ladder approximation quark propagator in the Landau gauge in Minkowski space that we use is $iS(k) = (\not{k} + M)(k^2 - M^2)^{-1}$, where M is a 2×2 quark mass function matrix assumed to be symmetric for this study. The ladder approximation quark propagator is accurate enough for our purposes because this is an exploratory study of a quark self-energy SDE which includes QCD and Yukawa contributions [18].

The gluon propagator we use is

$$g_s^2 G^{\mu\nu}(k) \equiv i \left(-g^{\mu\nu} + \frac{k^\mu k^\nu}{k^2} \right) G(k) \tag{2}$$

with

$$G(k) = \frac{g_s^2(k)}{k^2} \tag{3}$$

and the leading-log (one-loop) coupling

$$g_s^2(k) = \frac{4\pi^2 d}{ln(x_0 - \frac{k^2}{\Lambda_{QCD}^2})} \tag{4}$$

where $d = 12/(33 - 2n_f)$, n_f is the number of flavors. This form has the correct $-k^2 \to \infty$ leading order QCD behavior (for $-k^2 > 0$). This is a simple extension of the form used successfully to fit J/ψ and Υ spectroscopic data [19]. This leading-log QCD coupling is renormalization-scheme and gauge independent, as is its second order, two-loop, extension [20]. x_0 is a parameter that functions as a smooth infrared cutoff. Equation (4) does not accurately model the gluon potential in the infrared region, but it does enable us to assess the relative magnitude and shape effects of the Higgs boson and gluon exchanges in the quark self-energy [21].

The QCD coupling also contains the parameter n_f, the number of quark flavors. The number of quark flavors is determined by the energy scale, $y = \frac{q^2}{\Lambda_{QCD}^2}$, where q is the momentum transfer. We use a step-function expression to increment n_f at $q =$1.27 GeV ($n_f = 4$), 4.25 GeV ($n_f = 5$), 160 GeV ($n_f = 6$), and 500 GeV ($n_f = 8$), where the running quark

mass values $m_c(m_c) = 1.27$ GeV, and $m_b(m_b) = 4.25$ GeV are from Gasser and Leutwyler [22].

We must also model the Yukawa couplings. We consider a 2×2 symmetric Yukawa coupling matrix, which can be diagonalized by a 2×2 unitary matrix, to give diagonal elements $g_\pm$. The one-loop Renormalization Group Equation for the diagonalized Yukawa couplings, $g_\pm$, without weak interaction effects, is

$$\frac{dg_\pm}{dt} = \frac{1}{4\pi^2}\left(\frac{9}{8}g_\pm^3(t) - 2g_{QCD}^2(t)g_\pm(t)\right) \tag{5}$$

with $t = ln(x_0 + q^2/\Lambda_{QCD}^2)/2$. This has solution

$$g_\pm^2 = \frac{1}{C_\pm t^{2d} - \frac{9}{16\pi^2}\frac{t}{1-2d}} \tag{6}$$

where the $C_\pm$ must be determined by boundary conditions. We apply the boundary conditions $m_\pm^2(1 \text{ GeV}) = g_\pm^2(1 \text{ GeV})\text{v}^2/2$ for small (< 150 GeV) running quark masses, and $m_\pm^2(m_\pm) = g_\pm^2(m_\pm)\text{v}^2/2$ for large running quark masses, where $\text{v} \simeq 246$ GeV, and $m_+ = m_t$ and $m_- = m_c$, if we consider the 2×2 mass matrix to represent the second and third generations, for example. We chose $\Lambda_{QCD} = 0.18$ GeV to set the boundary values of $g_\pm$.

We convert Eq. (1) into 4-d spherical polar coordinates, and integrate over ϕ and χ analytically. To integrate over θ, we employ the so-called angle approximation [15] in the QCD sector and its analog in the Yukawa sector. The angle approximation is equivalent to keeping the first term in an expansion of the angular, θ, integral [24]. It predicts the same leading behavior for the ultraviolet asymptotic forms of M(y) as given by operator-product-expansion analysis in the pure QCD case [25]. Roberts and McKellar [16] showed that the angle approximation may be useful for a qualitative study of the quark SDE when the infrared behavior is expected to be smooth, as in our gluon model. We also make the approximation $M_H^2 \ll \Lambda_{QCD}^2(x + y - 2\sqrt{xy}cos\theta)$. This approximation is appropriate for the heavy quark cases that we study. For the light quark cases, this approximation exaggerates Higgs effects, thereby providing an upper bound to any new nonperturbative effects. For all input quark masses this Yukawa sector approximation holds in the asymptotic region where $y \gg 10^5$.

The resulting 1-d SDE is thus

$$
\begin{aligned}
M = M_0 + \Lambda_{QCD}^2 \int_0^y xdx \Big[& -d\, M \cdot (-x\Lambda_{QCD}^2 - M^2)^{-1} \cdot \frac{1}{yln(y+x_0)} \\
& + \frac{1}{32\pi^2} g_Y(y)\, M \cdot (-x\Lambda_{QCD}^2 - M^2)^{-1} \frac{1}{y} g_Y(y) \Big] \\
+ \Lambda_{QCD}^2 \int_y^\infty xdx \Big[& -d\, M \cdot (-x\Lambda_{QCD}^2 - M^2)^{-1} \cdot \frac{1}{xln(x+x_0)} \\
& + \frac{1}{32\pi^2} g_Y(x)\, M \cdot (-x\Lambda_{QCD}^2 - M^2)^{-1} \frac{1}{x} g_Y(x) \Big] .
\end{aligned} \qquad (7)
$$

We solve Eq. (7) by converting it into a second order differential equation (see Appendix) and applying a fourth-order Runge-Kutta subroutine. To ensure that the differential equation is equivalent to our integral equation, we must enforce some initial conditions. The QCD quark propagator integral SDE relates the values for the derivatives of the quark masses at the origin to the quark masses themselves. For the pure QCD 1-particle case, $\frac{dM}{dy}|_{y=0} = -\frac{\alpha_{QCD}(0)}{2M(0)\pi}$, where $\alpha_{QCD}(0) = \pi d/ln(x_0)$ [24]. The initial conditions for our QCD plus Higgs boson study must be consistent with this. For the two-quark-generation case, we assume the propagator quark mass function takes the form $M_{ij}(y) = M_{0\,ij} + M_{1\,ij}\, y$ near the origin, where M_{0ij} are the initial input quark masses, $M_q(0)$, and the forms of the first derivatives of the mass matrix elements, $M_{1\,ij}$, are given in the Appendix. The pure QCD sector tells us further that since we have included a nonzero bare quark mass (see Eq. 7), we must agree with the irregular QCD mass solution in the asymptotic region, i.e. for $y \gg 1$, $M(y) \propto (ln(y + x_0))^{-d}$ as long as QCD is the dominant effect [5]. We will use this as an asymptotic test of our numerical results.

quark	M_q(0 GeV)	m_q(1 GeV)
up	560 MeV	5.6 MeV
down	560 MeV	9.9 MeV
strange	720 MeV	199 MeV
charm	1.66 GeV	1.32 GeV
bottom	4.89 GeV	4.52 GeV

Table 1a: Input quark masses

Numerical input values are given in Tables 1.

quark	M_q(0 GeV)	$m_q(m_q)$
top	179 GeV	174 GeV
bottom′	506 GeV	500 GeV
top′	506 GeV	500 GeV

Table 1b: Input quark masses

We use two sets of input quark masses. For up, down, charm, strange, and bottom quarks at q=0 we use Jain and Munczek's [26] values as initial conditions on the $M_q(0)$ for the integration subroutine. The 1 GeV values are used to fix the values of $C_\pm$ of the Yukawa sector (except for the top quark and fourth generation quarks). The light quark (u,d,s) 1 GeV input masses are obtained by choosing values at 1 GeV which are consistent with the 1994 Particle Data Book and Y. Koide's paper "Table of Running Quark Masses" [27]. The heavy quark (c and b) 1 GeV mass values are obtained by running the heavy quark mass values given in the 1994 Particle Data Book to 1 GeV [28]. The Schwinger-Dyson evolution of $M(q)$ is small below $m_q(q)$ for heavy quarks [26], as seen from the slightly higher evolved values of $M_{t,t',b'}(0)$ starting from $m_q(m_q)$ values. For the top quark and the fourth generation quarks we normalize $C_\pm$ at $m_\pm(m_\pm)$ as indicated after Eq. (6).

For the multiple-quark-generation case we must also input the $q^2/\Lambda^2_{QCD} = 0$ values of the mixing angles for the up-sector and down-sector. We somewhat arbitrarily choose the up-sector mixing angle to be $\theta_{up} = 0.5$ radians. To ensure that the cabibbo mixing angle agrees with the accepted SM value for the first two generations, $\theta_{cabibbo} = 0.22$ radians at small q^2 [28], we choose $\theta_{down} = \theta_{up} - \theta_{cabibbo}$. For the third-fourth two-generation case, our convention is to adopt the same $\theta_{cabibbo}$ input value, since it is unknown. For the second-third two-generation case, we utilize the leading angle expansion term of the 1994 Particle Data Book's V_{cb}, $sin\theta_{23} \equiv 0.04$ [28]. We are primarily interested in the question of the running of the mixing angles, not their specific values, in any case. Next we turn to the results of this analysis.

III. Results

We solve the one-quark-generation version of Eq. (7) for seven cases of quark input masses given in Table 1 for the running quark mass functions. We summarize some characteristics of these results, including the initial and final values of the quark mass functions in Table 2, where A and B are the initial and final integration points, for example 5×10^{-5}, and 9.77×10^{8}, respectively, in units of Λ^2_{QCD}. We also compare the QCD plus Higgs boson exchange case with the QCD-only case in the asymptotic region to get the ratio $\frac{M_{QCD+H}}{M_{QCD}}$.

quark	$M_q(A)$	$M_q(B)$	$\frac{M_{QCD+H}}{M_{QCD}}$
up	$3.11\Lambda_{QCD}$	$1.00\Lambda_{QCD}$	1
down	$3.11\Lambda_{QCD}$	$1.00\Lambda_{QCD}$	1
strange	$3.99\Lambda_{QCD}$	$1.42\Lambda_{QCD}$	1
charm	$9.22\Lambda_{QCD}$	$4.21\Lambda_{QCD}$	1
bottom	$27.2\Lambda_{QCD}$	$15.3\Lambda_{QCD}$	$\simeq$1
top	$978\Lambda_{QCD}$	$825\Lambda_{QCD}$	1.01
bottom$'$=top$'$	$2790\Lambda_{QCD}$	$3140\Lambda_{QCD}$	$\neq$1, varies

Table 2: One-quark-generation results, for $A = 5 \times 10^{-5}$, $B = 9.77 \times 10^{8}$

We solve the second order differential equation version of Eq. (7), with initial conditions given in an Appendix, to get the quark mass function weak eigenstates M_{11}, M_{12}, M_{22}. We get the quark mass function eigenstates from $M_{\pm} = \frac{1}{2}\left[M_{11} + M_{22} \pm \sqrt{(M_{11} - M_{22})^2 + 4M^2_{12}}\right]$ and the mixing angles from $tan2\theta_f = 2M_{12}[(M_{22} - M_{11}) \pm \sqrt{(M_{22} - M_{11})^2 + 4M^2_{12}}]^{-1}$, where f stands for the flavor-sector, and where $\theta_{cabibbo} = \theta_{up} - \theta_{down}$. We look at three cases of two-quark-generations: first and second generations, second and third generations, and third and fourth generations.

In Figure 1, we graph the bottom quark mass function for the QCD plus Higgs boson case (solid line) and for the QCD-only case (dashed line) from the second and third generations analysis. The two lines are coincident. We also graph the top quark mass function for our QCD plus Higgs boson model (solid line) from the second and third generations analysis in Figure 2, with the corresponding QCD-only case (dashed line) shown for comparison. The

QCD plus Higgs boson case of the top quark mass function shows some deviation from pure QCD. The Higgs boson term has an opposite sign relative to the gluon term, so it drives the mass function up, in contrast to the usual QCD-only decreasing mass function. In Figure 3 we illustrate the dependence of the fourth generation top′ quark mass function on the energy scale, from the third and fourth generations case. The Higgs boson has a substantial effect on the fourth generation quark self-energy, i.e. the QCD plus Higgs boson case (solid line) disagrees substantially with the pure QCD case (dashed line). In fact, the running top′ quark mass is increasing in the asymptotic region. In Figure 4 we graph the 'running' Cabibbo mixing angle versus the energy scale for the third-fourth generations case, which shows the largest Higgs effect on the mass. There does not appear to be any variation in the mixing angle with respect to energy scale. This is true in all cases we study.

In Table 3 we summarize the two-quark-generation results. The two-quark- generation results are very similar to the one-quark-generation results, as we see by comparing Tables 2 and 3. Apparently the mixing effects in our model in the two-generation cases do not significantly affect the quark mass functions.

quark	$M_q(A)$	$M_q(B)$	$\frac{M_{QCD+H}}{M_{QCD}}$
up	$3.11\Lambda_{QCD}$	$1.00\Lambda_{QCD}$	1
down	$3.11\Lambda_{QCD}$	$1.00\Lambda_{QCD}$	1
strange	$3.99\Lambda_{QCD}$	$1.42\Lambda_{QCD}$	1
charm	$9.22\Lambda_{QCD}$	$4.21\Lambda_{QCD}$	1
bottom	$27.2\Lambda_{QCD}$	$15.3\Lambda_{QCD}$	1
top	$993\Lambda_{QCD}$	$840\Lambda_{QCD}$	1.01
bottom′=top′	$2810\Lambda_{QCD}$	$3150\Lambda_{QCD}$	$\neq$1, varies

Table 3: Two-quark-generation mass results, for $A = 5 \times 10^{-5}$, $B = 9.77 \times 10^{8}$.

IV. Conclusions

All calculational results have been verified by an independent calculation as indicated in the Appendix. We have calculated the momentum-dependent, quark propagator mass

for seven flavors of one-quark-generation quarks via a nonperturbative SDE treatment with Higgs boson exchange contributions. We do not see the effect in the ultraviolet region on the quark self-energy due to adding the Higgs boson interaction until the input quark mass is sufficiently large, $M_q(0 \text{ GeV}) > 75$ GeV. Thus only the top quark and the fourth generation quark results differ from pure QCD results.

We have also calculated the quark mass function for eight flavors of two-quark-generation quarks via a nonperturbative Schwinger-Dyson matrix equation analysis with Higgs boson exchange contributions to the quark self-energy. The results are similar to the one-quark-generation results. Only the top quark and fourth generation quarks, top$'$, and bottom$'$ differ significantly from the pure QCD results, with the fourth generation quarks showing the most marked effect of the Higgs. For the fourth generation quarks, the quark mass function actually increases with energy-scale, due to Higgs boson exchange.

We can analytically determine where the Yukawa term dominates. In terms of $y = q^2/\Lambda^2_{QCD}$ and $d = 12/(33 - 2n_f)$ we get

$$y > \left| exp\left\{ \left[\left(1 - \frac{9d}{2d-1}\right) \frac{(2)^{2d}}{32\pi^2 dC_\pm} \right]^{\frac{1}{2d-1}} \right\} - x_0 \right|$$

where $C_\pm < 0$ for $y > -x_0 + exp\{\frac{16\pi^2 \text{v}^2}{m^2_\pm} \frac{2d-1}{d}\}$. Thus for the top quark, $C_t = -0.19$, and the Yukawa term dominates when q is beyond the Planck scale. For the fourth generation quark, $C_{4th} = -0.053$, and the Yukawa term dominates when $q > 1.5$ TeV.

We have calculated the running mixing angle, which we call theta-cabibbo, for three cases of two-quark-generations. We detect no running of theta-cabibbo or any other mixing angle due to any effect, for any quark mass input value up to and including 500 GeV. This is expected for pure QCD and can be proven analytically [29]. This provides a check of the numerical work. By studying the mixing angle results, we determine that the running of the mixing angles due to nonperturbative effects in our model is less than one part in 10^9, far less than any conceivable experimental sensitivity. We have been unable to prove analytically that the cabibbo angle is constant when the Yukawa interaction is included in the SDE analysis, but we have established numerically that there can be no observable effect even for the heavy hypothetical fourth generation. Based on our results, it appears to be safe to ignore running of V_{ts} and V_{tb}, for example, in extrapolating from m_b^2 to m_t^2 in parametrizing

data [1, 30]. Our nonperturbative results agree in this respect with the RGE analysis of the model that we study, since the RGE's can always be diagonalized completely and no running of the mixing angles occurs. As remarked earlier, the y-dependence of our mass functions agrees with that of the 1-loop RGE's in the asymptotic region, though not in the region at or below the scale of the input masses.

In our investigation we have neglected the contributions of Goldstone bosons, $W^{\pm}$, Z^0 and γ exchange in the quark self-energy. Of these, we expect only the Goldstone bosons to have a large effect on the quark self-energy in the Landau gauge, and then only in the heavy quark sector where it is strongly coupled. Thus the next step in this analysis is to expand our model to include Goldstone boson exchange. For heavy quarks, we expect the effects on the quark self-energy from the electroweak bosons to be significantly smaller than those of the gluon, Higgs boson, and Goldstone bosons.

We could also expand the flavor sector to include the three-quark- generation case and possibly the four-quark-generation case. This would enable us to calculate the running of the elements of the three-quark-generation Cabibbo-Kobayashi-Maskawa matrix and its four-quark-generation analog. These calculations should be undertaken with due consideration, however, since introducing additional quark generations significantly increases the length of the calculations. Moreover, our two-quark-generation case seemed to indicate that the effects on running quark mass functions due to mixing between generations are negligible.

This method is well-suited to study a variety of nonperturbative non-Standard-Model effects. For example, we could investigate the case of the third generation top quark experiencing a new gauge interaction instead of, or in addition to, the Higgs mechanism by replacing the Higgs boson exchange with a new gauge boson exchange in the top quark self-energy. This would enable us to study models of dynamical breaking of electroweak symmetry where the top quark plays a special role [31, 32]. Alternately, we could study the case of the fourth generation quarks experiencing a new gauge interaction, in addition to the Higgs mechanism, which the SM quarks do not experience, to study new, dynamical effects. We address these and other, related, points in a future publication.

Acknowledgements

L.L.S. thanks Craig Roberts for discussions and for sponsorship under the “Thesis Parts” program at Argonne National Laboratory. D.W.M. thanks the Department of Physics at U.C. Davis, and especially Ling-Li Chau, Jack Gunion and Barry Klein, for hospitality during his sabbatical leave while this work was in progress. L.L.S. was supported in part by a DOE Traineeship and by the University of Kansas General Research Fund. This work was supported in part by the DOE Grants No.s DE-FG02-85ER-40214 and DE-FG05-91ER-40636.

Appendix

Using $g_\pm$ we can specify g_{ij}:

$$g_{11}^2 = g_+ sin^2\theta + g_- cos^2\theta, \tag{8}$$

$$g_{12}^2 = (g_+ - g_-) cos\theta\, sin\theta, \tag{9}$$

$$g_{22}^2 = g_+ cos^2\theta + g_- sin^2\theta. \tag{10}$$

Then in terms of

$$\mathcal{A} = \frac{d}{y^2 ln(x_0 + y)} - \frac{g_{11}^2 + g_{12}^2}{32\pi^2 y^2}, \tag{11}$$

$$\mathcal{B} = \frac{g_{12}(g_{11} + g_{22})}{32\pi^2 y^2}, \tag{12}$$

$$\mathcal{C} = \frac{d}{y^2 ln(x_0 + y)} - \frac{g_{12}^2 + g_{22}^2}{32\pi^2 y^2}, \tag{13}$$

and

$$\begin{aligned} \mathcal{D} &= (y\Lambda_{QCD}^2 + M_{11}^2 + M_{12}^2)(y\Lambda_{QCD}^2 + M_{12}^2 + M_{22}^2) \\ &- M_{12}^2 (M_{11} + M_{22})^2, \end{aligned} \tag{14}$$

$$\mathcal{M}_{11} = M_{11}(-y\Lambda_{QCD}^2 - M_{22}^2) - M_{12}^2 M_{22}, \tag{15}$$

$$\mathcal{M}_{12} = M_{11} M_{12} M_{22} + M_{12}(-y\Lambda_{QCD}^2 - M_{12}^2), \tag{16}$$

$$\mathcal{M}_{22} = M_{12}^2 M_{11} + M_{22}(-y\Lambda_{QCD}^2 - M_{11}^2), \tag{17}$$

we can write the components of the second order differential equation:

$$\frac{d^2M_{11}}{dy^2} = \frac{\Lambda_{QCD}y}{\mathcal{D}}(\mathcal{M}_{11}\mathcal{A} - \mathcal{M}_{12}\mathcal{B})$$

$$+\frac{1}{\mathcal{A}\mathcal{C}-\mathcal{B}^2}\left(\frac{dM_{11}}{dy}(\mathcal{E}\mathcal{C}+\mathcal{F}\mathcal{B}) + \frac{dM_{12}}{dy}(\mathcal{F}\mathcal{C}+\mathcal{G}\mathcal{B})\right), \quad (18)$$

$$\frac{d^2M_{12}}{dy^2} = \frac{\Lambda_{QCD}y}{\mathcal{D}}(-\mathcal{M}_{11}\mathcal{B} + \mathcal{M}_{12}\mathcal{C})$$

$$+\frac{1}{\mathcal{A}\mathcal{C}-\mathcal{B}^2}\left(\frac{dM_{11}}{dy}(\mathcal{E}\mathcal{B}+\mathcal{F}\mathcal{A}) + \frac{dm_{12}}{dy}(\mathcal{F}\mathcal{B}+\mathcal{G}\mathcal{A})\right), \quad (19)$$

$$\frac{d^2M_{22}}{dy^2} = \frac{\Lambda_{QCD}y}{\mathcal{D}}(-\mathcal{M}_{12}\mathcal{B} + \mathcal{M}_{22}\mathcal{C})$$

$$+\frac{1}{\mathcal{A}\mathcal{C}-\mathcal{B}^2}\left(\frac{dM_{12}}{dy}(\mathcal{E}\mathcal{B}+\mathcal{F}\mathcal{A}) + \frac{dm_{22}}{dy}(\mathcal{F}\mathcal{B}+\mathcal{G}\mathcal{A})\right), \quad (20)$$

where

$$\mathcal{E} = -\frac{2\mathcal{A}}{y} - \frac{d}{y^2 ln^2(x_0+y)\,(x_0+y)} \quad (21)$$

$$\mathcal{F} = \frac{2\mathcal{B}}{y} \quad (22)$$

$$\mathcal{G} = -\frac{2\mathcal{C}}{y} - \frac{d}{y^2 ln^2(x_0+y)\,(x_0+y)}. \quad (23)$$

The initial condition $M_{1\,ij}$ are given by:

$$\frac{dM_{11}(y)}{dy}|_{y=0} = -\frac{\Lambda^2_{QCD}y^2}{2\cdot\mathcal{D}}(\mathcal{M}_{11}\mathcal{A} - \mathcal{M}_{12}\mathcal{B}) = M_{1\,11} \quad (24)$$

$$\frac{dM_{12}(y)}{dy}|_{y=0} = -\frac{\Lambda^2_{QCD}y^2}{2\cdot\mathcal{D}}(-\mathcal{M}_{11}\mathcal{B} + \mathcal{M}_{12}\mathcal{C}) = M_{1\,12} \quad (25)$$

$$\frac{dM_{22}(y)}{dy}|_{y=0} = -\frac{\Lambda^2_{QCD}y^2}{2\cdot\mathcal{D}}(-\mathcal{M}_{12}\mathcal{B} + \mathcal{M}_{22}\mathcal{C}) = M_{1\,22}. \quad (26)$$

In [29] we use $S^{-1} = -i[\not{q}A - B]$ and solve the integro-differential system

$$A^{-1}(q^2) = 1 + \frac{i}{q^2}\int\frac{d^4k}{(2\pi)^4}k\cdot q\, g_Y[k^2 - M^2(k^2)]^{-1}\, g_Y\frac{1}{(k-q)^2 - M_H^2}$$

$$-\frac{i}{q^2}\int\frac{d^4k}{(2\pi)^4}\left[-(1+\xi)k\cdot q + 2(\xi-1)\frac{q\cdot(k-q)k\cdot(k-q)}{(k-q)^2}\right]$$

$$\times [k^2 - M^2(k^2)]^{-1} \frac{4\pi\alpha_s}{(k-q)^2} \quad (27)$$

$$M(q^2) = M_0 A^{-1} + i \int \frac{d^4k}{(2\pi)^4} g_Y \, M[k^2 - M^2(k^2)]^{-1} g_Y \frac{1}{(k-q)^2 - M_H^2}$$

$$- i(3+\xi) \int \frac{d^4k}{(2\pi)^4} M[k^2 - M^2(k^2)]^{-1} \frac{4\pi\alpha_s}{(k-q)^2} \;, \quad (28)$$

where ξ is the gauge parameter. Our gauge choice here is $\xi = 0$. Equations (27) and (28) are gauge independent at large momenta where the terms depending on ξ are subleading. We have solved Eqs. (27) and (28) by using the angle approximation for the QCD terms, taking g_Y as a constant, and performing the angular integrals exactly for the Higgs term with a massive Higgs boson. The result is converted into integro-differential equations and solved numerically. The solutions for the mass functions agree very well with solutions to Eqs. (18)-(20) for a variety of boundary conditions. Further applications of the $A \neq 1$ solutions are presented in [29].

References

[1] CLEO Collaboration (Jesse Ernst *et. al.*), UR-1381A. Talk given at a 1994 Meeting of the American Physical Society, Division of Particles and Fields (DPF94), Albuquerque, NM, 2-6 Aug. 1994.

[2] Y. Nambu, G. Jona-Lasinio, *Phys. Rev.* **122** (1961) 345.

[3] M.E. Peskin, in *Recent Advances in Field Theory and Statistical Mechanics*, eds. J.B. Zuber, R. Stora (Elsevier, 1984).

[4] P.I. Fomin, V.P. Gusynin, V.A. Miransky, Yu. A. Sitenko, *Rivista del Nuovo Cimento* **6**, Series 3, No. 5 (1983).

[5] D. Atkinson, P.W. Johnson, *Phys. Rev.* **D35** (1987) 1943; *ibid.* **D37** (1988) 2290; *ibid.* (1988) 2296.

[6] J.M. Cornwall, R. Jackiw, E. Tomboulis, *Phys. Rev.* **D10** (1974) 2428; V.A. Miransky, P.I. Fomin, *Phys. Lett.* **105B** (1981) 387; D. McKay, H. Munczek, *Phys. Rev.* **D42** (1990) 3548.

[7] H. Pagels, *Phys. Rev.* **D14** (1976) 2747;
ibid. **D15** (1977) 2991;
J.M. Cornwall, *ibid.* **D22** (1980) 1452;
J.M. Ball, F. Zachariasen, *Phys. Lett.* **106B** (1981) 133;
H.J. Munczek, *Phys. Lett.* **B175** (1986) 215.

[8] H. Pagels, S. Stokar, *Phys. Rev.* **D20** (1979) 2947.

[9] R. Jackiw, K. Johnson, *Phys. Rev.* **8** (1973) 2386.

[10] H.J. Munczek, D.W. McKay, *Phys. Rev.* **D39** (1989) 888.

[11] A. Barducci, R. Casalbuoni, S. DeCurtis, D. Dominici, R. Gatto, *Phys. Rev* **D38** (1988) 238.

[12] C.D. Roberts, A.G. Williams, *Dyson-Schwinger Equations and their Applica tion to Hadronic Physics*, in *Progress in Particle and Nuclear Physics*, ed. A Fabler (Pergamon, 1994).

[13] A. Hadicke, *Intern. J. Mod. Phys.* **A6** (1991) 3321.

[14] C.D. Roberts, *Schwinger-Dyson Equations: Dynamical Chiral Symmetr Breaking and Confinement, in QCD Vacuum Structure*, eds. H.M. Fried, B Muller (World Scientific, 1993).

[15] K. Higashijima, *Phys. Lett.* **B124** (1983) 257;
Phys. Rev. **D29** (1984) 1228.

[16] C.D. Roberts, B.H.J. McKellar, *Phys. Rev.* **D41** (1990) 672.

[17] For a discussion see K. Kondo, H. Nakatani, *Mod. Phys. Lett.* **A4** (1989) 2155 (1990) 672.

[18] T. Appelquist *et. al.* have determined that next-higher -order terms beyond the ladder approximation amount to a 1%-20% correction, depending on the fermion representation, for theories in which the coupling is a slowly running

function of momentum and is large enough to trigger spontaneous chiral sym-
metry breaking.
T. Appelquist, K. Lane, U. Mahanta, *Phys. Rev. Lett.* **61** (1988) 1553.

[19] J.L. Richardson, *Phys. Lett.* **B82** (1979) 272.

[20] F.J. Yndurain, *Phys. Lett.* **63B** (1976) 211;
E. Reya, *Phys. Rep.* **69** (1981) 195;
G. Altarelli, N. Cabibbo, G. Corbó, L. Maiani, G. Martinelli, *Nucl. Phys.* **B208**
(1982) 365.

[21] We choose a median value, $x_0 = 5$, in comparison with Munczek and Jain's $x_0 = 10$ [26], and D. Atkinson *et. al.*'s values $1 \geq x_0 \geq 7.3$ [5], for example. We calculate, moreover, that the running quark mass function does not depend very strongly on x_0. The $x_0 = 5$ and $x_0 = 10$ cases vary by 1.6% and 2.6% respectively, from the $x_0 = 2$ value.

[22] J. Gasser, H. Leutwyler, *Phys. Rep.* **87** (1982) 77.

[23] P. Jain, A.J. Sommerer, D. W. McKay, J.R. Spence, J.-P. Vary, B.-L. Young, *Phys. Rev.* **D49** (1994) 2514.

[24] D. McKay, H.J. Munczek, *Phys. Rev.* **D42** (1990) 3548.

[25] H.D. Politzer, *Nucl. Phys.* **B117** (1976) 397.

[26] H.J. Munczek, P. Jain, *Phys. Rev.* **D46** (1992) 438.

[27] Y. Koide, University of Shizuoko Report No. US-94-05, October 1994.

[28] Particle Data Group, L. Montanet *et. al.*, *Phys. Rev.* **D50** (1994) 1173.

[29] L.L. Smith, P. Jain, D.W. McKay, manuscript in preparation.

[30] We thank Dave Besson for emphasizing this point to us.

[31] W.A. Bardeen, C.T. Hill, M. Lindner, *Phys. Rev.* **D41** (1990) 1647;

Y. Nambu, in *New Theories in Physics, Proceedings of the XI Internationa Symposium on Elementary Particle Physics*, eds. Z. Ajduk, S. Pokorski, A Trautman (World Scientific, 1989).

[32] V.A. Miransky, M. Tanabashi, K. Yamawaki, *Mod. Phys. Lett.* **A4** (1989) 1043

Figure Captions

Figure 1: $m_{bottom}(y)/\Lambda_{QCD}$ vs. y for $A = 5 \times 10^{-5}$, $B = 9.77 \times 10^{8}$, and inputs $x_0 = 5$, m_0 =4.89, 506 GeV. The solid (dashed) line is the gluon plus Higgs boson (QCD-only) interaction result.

Figure 2: $m_{top}(y)/\Lambda_{QCD}$ vs. y for $A = 5 \times 10^{-5}$, $B = 9.77 \times 10^{8}$, and inputs $x_0 = 5$, m_0 =179, 506 GeV. The solid (dashed) line is the gluon plus Higgs boson (QCD-only) interaction result.

Figure 3: $m_{top'}(y)/\Lambda_{QCD}$ vs. y for $A = 5 \times 10^{-5}$, $B = 9.77 \times 10^{8}$, and inputs $x_0 = 5$, m_0 =179, 506 GeV. The solid (dashed) line is the gluon plus Higgs boson (QCD-only) interaction result.

Figure 4: $\theta_{cabibbo}(y)$ vs. y for $A = 5{\times}10^{-5}$, $B = 9.77{\times}10^{8}$, and inputs $x_0 = 5$, m_0 =4.89, 506 179, 506 GeV. The solid (dashed) line is the gluon plus Higgs boson (QCD-only) interaction result.